STOCHASTIC ANALYSIS OF QUEUING SYSTEMS WITH BATCH ARRIVALS, SINGLE / MULTIPLE VACATIONS

A SYNOPSIS

Submitted by

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1. INTRODUCTION

Queuing theory is a mathematical analysis of queuing. Basically, a queue is a system that functions in the following way: Input, Service mechanism and output. Queues form when there is an opportunity to provide a service that exceeds the capacity to provide that service. Queuing theory helps you to make better decisions. It helps you to maximize system efficiency. Examples for Queues: Machines waiting for repairs, patients waiting in hospital, automobiles waiting at traffic, customers waiting in ticket counters, and people waiting in banks and post offices.

DESCRIPTION OF THE QUEUING PROBLEM

Customers entering the system in search of service, waiting for it if it is not available right away, and then exiting the system once they have received it can be defined as a queuing system. This is explained in Figure 1.1

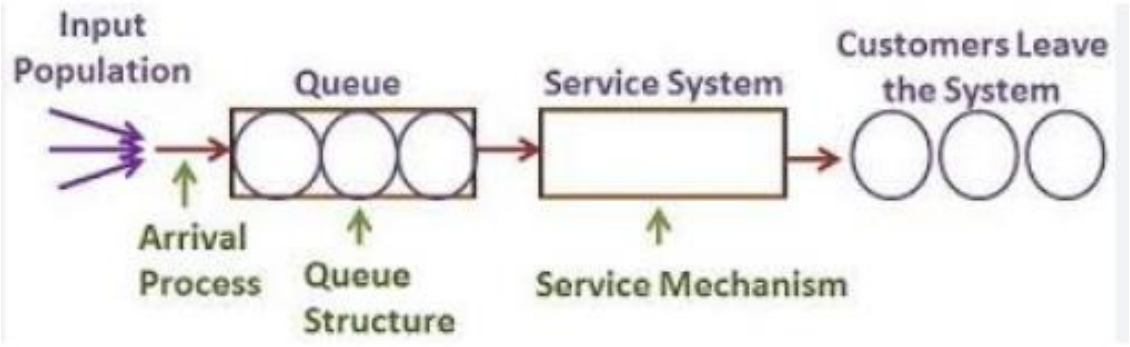

Figure 1.1 Queuing System

CHARACTERIZATION OF QUEUING SYSTEM

A queuing system can be classified and described according to the following basic six characteristics:

Arrival pattern (or input pattern)

The way clients arrive for service and enter the queuing system is described by the arrival (input). It is determined by the time difference between any two successive arrivals. They arrive in single or batches.

The arrival rate is the number of customers arriving per unit time and is represented by the symbol λ. The arrival rate follows a Poisson distribution with mean λ.

Service pattern (or Departure pattern)

The length of time needed to provide a service for a consumer determines the service time. The term "service rate" refers to the quantity of clients serviced per unit of time and is represented by the symbol μ. The inter-service time follows an exponential distribution with mean $\frac{1}{\mu}$.

Service Facility

The system might only have one service channel. For the purpose of offering clients an identical service, the system may have multiple servers set up in parallel, series, or a combination of both.

System capacity

Some queuing procedures have physical limits on the amount of waiting space. Thus there is a finite limit to the maximum system size. We can state that the system has infinite capacity if any number of clients are permitted to join the line.

Queue discipline

The process of choosing customers to receive service is called queue discipline. The most popular disciplines are: First Come First Service (FCFS), Last Come First Service (LCFS), Service in Random Order (SIRO), Priority service.

Service stages

The number of service stages in a queue system might range from one to many. In multistage queuing systems, a customer enters one queue, waits for service, is serviced, leaves the service station, and then enters another waiting for yet another service, and so on. It may be possible to allow recycling or feedback in some multistage queuing systems.

CUSTOMER BEHAVIOUR

Customers typically act in the following ways:

(i) **Balking:** A customer is said to be baulking if they refuse to join the queue because it is too long.

(ii) **Reneging:** A customer is considered to have reneged if they join the queue and then leave the queue before obtaining service because they were impatient.

(iii) **Jockeying:** A customer who jumps from one queue to another with a shorter length to shorten the wait time is considered to be jockeying.

(iv) **Retrial:** A retrial queue is a model of a system with finite capacity in which tasks that arrive and find the system busy wait for a period of time before attempting to join the system again.

LITTLE'S FORMULA

In the early 1960s, John D.C. Little created one of the queuing theory's most potent correlations. He established a connection between the average customer waiting time in the system and the mean number of customers in the system. Little's law states that "the mean number of customers in a queuing system is equal to the mean arrival rate of customers to that system times the mean time spent in that system". That is $L = \lambda W$

POISSON DISTRIBUTION

Poisson distribution is a discrete random variable. It involves in the probability of occurrence of an event. The average arrival rate λ is assumed to be constant over time and is independent of the number of units already arrived.

EXPONENTIAL DISTRIBUTION

Exponential distribution is a continuous random variable. It involves the probability of completion of a service. The average service rate μ is assumed to be constant over time and independent of number of units already serviced.

In reality, the exponential distribution occurs in waiting time issues. Exponential distribution follows an important property namely memory less property

2. LITERATURE REVIEW

A number of authors have made brief observations on the Non-Markovian model. Long waits, or queues, are an aspect of queuing theory.

Analysis of a bulk queue with unreliable server was analyzed by Ayyappan G. and Karpagam S. [1]. Chen X et al. [3] displayed an investigation of swarm intelligence techniques in VLSI routing problems. Jain. M et al. [5] discussed in detail about the Supplementary variable technique for Non Markovian single server queue with service interruption. Jayapradha et al. [6] studied the Detection and Diagnosis of Faults in VLSI Testing. A study on the performance metrics of the Non-Markovian model of optional forms of service with extended vacation, reneging process, and service disruption followed by phases of repair process was provided by Maragathasundari S and Manikandan P [9]. Maragathasundari. S and Srinivasan.S [10] have studied the effect on optional services in a non-commercial queue. The queuing issue on a Whats App server is solved by means of a queuing approach, and the analysis was thorough by Maragathasundari S. et al. [11]. Shanmugasundaram and Sivaram [12] analyzed the M/G/1 Feedback Queue when the server is off and on vacation. Pakkirisami Rajadurai [13] investigated the process of performance analysis of an M/G/1 retrial G-queue with feedback under working breakdown services. In a Non- Markovian queuing model, Singhal et al. [15] looked atthe customer's waiting time value for general queues.

All of the queuing literature described above contains very detailed information on the vacation time. The perspectives of many authors regarding the Vacation approach as a required step have been expressed in a wide range of publications. Additionally, the break doesn't start until all service steps are complete. However, no author has yet added a vacation to their work during the intervals in between services. In many real-life scenarios, a pause during the service is essential. In this case, a multi-vacation plan is also helpful. In this study, the concepts of flexible vacation, Reneging, phases of services, interruptions, and fixation process have all been introduced together with the investigation of queuing technique.

3. OBJECTIVES

In previous work on the Non -Markovian queue, our researchers focused on a single vacation to a great extent rather than a multiple vacation. Also, service is only provided at certain stages. Service phases and the process of delay have not been well studied.

In this study, we assume a $M^{[X]}/G/1$ queuing model The numerous assumptions included in our models differ from previous work in terms of stages of service, the idea of reneging, setup time stages of service, delay process, restricted admissibility, multiple vacation policy of long vacation and short vacation.

This study is conducted with the following objectives:

1. A study on Non-Markovian queuing problem of different setup time stages of service and optional vacation.
2. An investigation on a batch arrival queuing model with multi vacation policy of restricted admissibility and standby server.
3. An examination on a different stages of service with service interruption and optional vacation.

The Queuing models examined in the present study are able to address many real-world dilemmas that occur in a system, such as when the servers are not continuously available to provide service to arriving customers; when the server is under maintenance work and at the time of customer reneging, the repair process, the introduction of setup time stages, and the multi vacation policy. These findings give granular data for large-scale enterprises, communication networks, and other domains where making judgments on queued systems is critical. As a result, we anticipate that this research will considerably assist system designers in making decisions about system parameters, and that our work will contribute new information to queuing theory.

4. RESEARCH PROBLEM

In our work, we discussed with the following research problem:

- Analysis on performance metrics of a Non-Markovian queuing problem of an avionic communication.
- A study on a Non-Markovian boundary value problem's organizational and queue description.
- Assessment of performance simulation in VLSI testing procedures.

The aforementioned issues are addressed in our research work. The queuing challenges identified in our paper play an important role in multi-tier systems such as e-commerce, motor manufacturing, vehicle manufacturing, and textile manufacturing. The specified problem, in particular, plays a key role in avionic communication and VLSI testing processes. The influence of various factors on the given queuing problem is investigated using R and MATLAB Soft wares. The brief explanation of our problems are given below:

Chapter 1 of this research gives a detailed explanation about the defined parameters in the queuing problem discussed. Next, the work focused on a queuing system in chapter 2 where the server enters the setup phase twice before the main service begins. The first start of the second setup timer is similarly delayed. These two setup phases complete all of the preliminary work for the main service. Service is disrupted due to unforeseen circumstances. The repair procedure began instantly and without delay, but the client was impatient and queued before leaving the machine unserved. This is also a reasonable assumption. As a result, at the completion of the service, a vacation procedure is established in order to minimize the disruption of this sort of service or to launch it without disrupting the system. If there are no consumers on the system once the service is terminated, the server enters a vacation phase and undertakes extensive server maintenance. As a result, the purpose of this research is to discover the steady-state behavior of queuing systems that have failed throughout the general vacation schedule, random failures, urgent repair process, and repair process phases. This paradigm was created to aid with avionics communication. As a result, we viewed the same structure as

an application in this post. Supplemental variable techniques are used to evaluate the performance measures of such queuing systems that occur in avionics communication. Numerical diagrams and visual frameworks help to analyze the model.

In chapter-3 of queuing system, a single server serves each client after they join up for the service. Customers wait in line for the second step of service after completing the first round of administration. The server takes a brief vacation to complete the necessary server maintenance after providing the first stage of service to all arriving clients and if there are no clients waiting in line for the first level of service. The server takes a long vacation that is split into two halves after the short vacation is over. However, the first step of extended vacation is necessary, although the second stage is not. The above description of the process' maintenance requirements is in-depth. Additionally, in order to reduce lengthy waits for procedures during brief vacations, the concept of restricted admissibility is established. Additionally, a backup server is accessible in case the server takes a second stage of an extended vacation. To prevent the line from growing too long, this is a necessary step in the process. As a result, a standby server will supply the second stage of service if the primary server enters the second stage of the prolonged vacation procedure. Such a type of queuing problem is solved by the supplementary variable technique and the performance measures are derived.

In Chapter-4, the study provides a piling strategy to address the significant issue with the VLSI testing methodology of service systems, including maintenance work, stages of service, setup time stage work, Reneging, interruption, and repair process. At various levels of VLSI testing methods, the insulating device is subject to probability movement. The strategy for VLSI testing has fully changed into a line problem. It involves providing a variety of administrations. In this context, the term "vacations" refers to the system maintenance work that is done as an option following the conclusion of stage 3 service. The system benefits as much as it can from this maintenance effort to provide a seamless service. This excursion can be taken or left out. Following the vacation, the server moves into a fourth level of service, where the last bit of work for the VLSI testing technique of services will be finished. Likewise, server failure occurs and is inevitable during stage 1 service. As a matter of fact, the server starts the repair procedure right away. Once the issue has been resolved, service is again made available. After the first phase of

service is finished, users wait in line to acquire stage 2. After finishing all service procedures, customers finally leave the system.

We employ the supplementary variable strategy in our study to address the described queuing issue, and as a consequence, the appropriate probability generating function of queue size and all other performance metrics of the queuing system are generated.

5. ORGANIZATION OF THESIS

CHAPTER 1

INTRODUCTION

1.1 MORKOVIAN AND NON-MORKOVIAN QUEUES

A queuing system in which service time follow an exponential distribution is called Markovian queuing system.

A queuing system is said to be Non-Markovian if the service time does not follow an exponential distribution.

1.2 DESCRIPTION OF THE M/G/1 AND $M^{[X]}$/G/1 SYSTEM

M/G/1 is a single server Non-Markovian queuing system in which arrivals follow a Poisson process and service time follows general distribution.

$M^{[X]}$/G/1 Queuing system differs from the above model only in the arrival. In this system, customers arrive to the system in batches of variable size. These models have been extensively due to its wide applications. The queue discipline is service in the order of arrival between batches, which means that all customers in the n^{th} batch are served before the first customer in the $(n+1)^{th}$ batch.

1.3 QUEUING SYSTEM CHARACTERISTICS

1.3.1 Service Interruption (σ)

A service interruption is a transmission interruption that completely eliminates the service's signal, making the service inoperable. The service will be "Available" if there

isn't a service interruption.

1.3.2 Random Breakdowns

In queuing systems where the server is a machine, such as networks, communication systems, and computer systems, it is realistic to assume that the server may unexpectedly go offline and won't be able to resume service until it is repaired. The breakdowns occur randomly during the time of rendering service and repair timeframes are dispersed in this queue system. Break down arrival rate follows Poisson distribution.

1.3.3 Repair Process (η)

The process of restoring a damaged or malfunctioning object to good working condition is referred to as the repair process. Whenever the system breaks down, its repair starts immediately. The repair time could follow an exponential, general, deterministic etc. distribution.

1.3.4 Setup Time Stages (β)

The amount of time needed to configure a machine so that it is prepared to handle a job is known as setup time. Whenever a service to be provided, it will be in need of a preprocessing work. Those necessary preprocessing work will be carried out in these set up time stages.

1.3.5 Reneging (φ)

Reneging is the act of a customer's leaving a queuing system without obtaining service after joining the queue because there was too much waiting (or because they were impatient).

1.3.6 Server Vacation (μ)

A vacation is a period of time whenever the servers are not accessible to just provide service in a queuing situation. The server will not service customers (inputs) that arrive during the vacation; instead, they will wait until the server returns from vacation. During the vacation period a bulk of the essential maintenance work to the server will be done. This helps the system to run without interruption or guarantee that the server runs constantly are referred to in this sense as vacations. Furthermore, after the end of the service, if there are no customers in queue and all arriving customers have been served then

the server goes on vacation. Upon return from a vacation, the server starts service.

1.3.7 Restricted Admissibility (p)

Whenever the service is provided to all the arriving customers, it is impossible at all times to allow all the arriving customers to get into the system for the service. Suppose due to various reasons (Maintenance work, Service interruption etc.,), if there is a long queue inside the system then many customers stand in a queue and this leads to a congestion in the system. To avoid such situation, there is a necessity to stop the customers at the entrance of the queuing system. The major customers who are in urgency will be taken care at first. This process is defined to be restricted admissibility. Restricted admissibility occurs during Vacation stages.

1.3.8 Standby Server (ϕ)

Several queuing models have practical uses, such as serving as a backup server when the primary server is unavailable (due to absences like vacations and server failures). A standby server is used in place of the primary server to ensure that clients receive uninterrupted service. This backup server is assumed as the primary server operating (at a lower rate) while off-site or undergoing maintenance. The arrival rate of standby servers is based on the Poisson distribution.

1.4 SUPPLEMENTARY VARIABLE TECHNIQUE (Method of solving queuing model)

This technique was made by David George Kendall and David Cox (1955), in which the system is made Markovian by introducing some supplementary variable elements. According to the exponentially distributed inter-arrival and service times of the M/M/1 queue, it is possible to model the queue size process N(t) by a Markov process. The arrival process and/or the service process of non Markovian queuing model fails to be memory less so we cannot model the queue size process N(t) by a Markov process. As a result, we introduce X as a new random variable (t). It displays the customer's length of service at time "t." By incorporating additional variables in the state description, we show how the two-dimensional stochastic process (N (t), X (t)) converts into a Markovian process. This is called Supplementary variable Technique.

CHAPTER 2

ANALYSIS ON PERFORMANCE METRICS OF A NON-MARKOVIAN QUEUING PROBLEM OF AN AVIONIC COMMUNICATION

2.1 INTRODUCTION

In this queuing system the clients (input) appear into the system in batches. Their arrival follows the Poisson ascension. Service time follows general distribution. The service here is provided in a single management phase. Before the main service, all clients must go through the two preparation phases of server management. Second stage pre-processing management is given after a delay. During this period, customers must queue up to receive service. Also an idea of reneging and restricted admissibility is proposed. To minimize the service interruption, vacation concept is introduced after the completion of server. This type of queuing situation is solved through supplementary variable technique. Probabilistic generating function of the queue size, idle time of the server, utilization factor and various execution measures are derived.

2.2 DIAGRAMMATIC REPRESENTATION OF THE MODEL

The defined queuing problem is well explained by the Figure 2.1

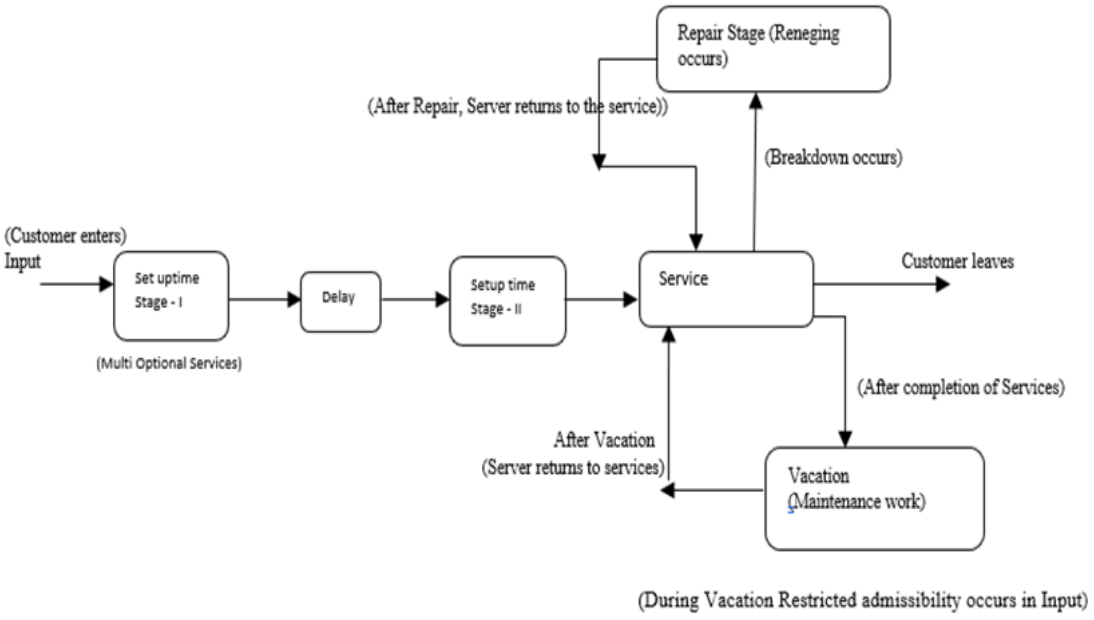

Figure 2.1 Diagrammatic Representation of the Model

2.3 DESCRIPTION OF THE DEFINED QUEUING MODEL

We investigated a new system that uses the idea of two servers with time stages and reneging new research concept. Therefore, this study is to identify steady-state behavior of queuing systems using batch arrival queue systems that have failed during the general vacation schedule, random failures, immediate repair process, and repair process phases. It was carried out. In addition, if the system does not have clients, the server may take a vacation and stay on the system to serve incoming clients. Two phases of setup time service are provided before the start of the main service. This allows the server to run smoothly without delay. In addition, there is a delay before the second level of setup service works. This delay is due to the preprocessing of the second setup timer work performed by the server.

The queuing system also introduces vacations after the service ends. Major maintenance work for the server will be done during this vacation. This allows the system to serve all incoming customers without interruption or with minimal interruption events. Amongst this busy schedule of the lining framework, even if the server gets interrupted, immediately the server was sent to repair the measurements and when the repair cycle was complete the server started maintenance. However, to minimize service interruptions, the server is sent during the vacation period (maintenance work) immediately after the end of service. The interruption was minimal, but a non-compliance occurred during the remediation process after server downtime. This means the customers leave the structure without getting the service due to impatience.

2.4 PERFORMANCE MEASURES OF THE QUEUING MODEL

By using supplementary variable technique the probability generating function of the queue size which forms a basics in finding the performance measures of the queuing model is derived.

$$R_q(z) = \frac{\{H\lambda_k\,(D(z)-1)\}\left\{\begin{array}{c}\left[\frac{1-S_1^*(a)}{a}\right]+S_1^*(a)\left[\frac{1-W^*(a)}{a}\right]+S_1^*(a)W^*(a)\left[\frac{1-S_2^*(a)}{a}\right]+\\ W^*(a)S_1^*(a)S_2^*(a)\left[\frac{1-A_1^*(b)}{b}\right]+W^*(a)S_1^*(a)S_2^*(a)A_1^*(b)\left[\frac{1-Y_1^*(c)}{c}\right]+\\ \sigma W^*(a)S_1^*(a)S_2^*(a)\left[\frac{1-B^*(d)}{d}\right]\end{array}\right\}}{z-W^*(a)S_1^*(a)S_2^*(a)\left[1+B_q{}^*(d)\sigma\left[\frac{1-A_1^*(b)}{b}\right]\right]}$$

Using Little's formula the other performance measures are derived.

(i) Idle time of the server $H = 1 - R_q(1)$

(ii) Utilization factor $\rho = 1 - H$

(iii) Average number of customers in the queue $L_q = \lim_{z \to 1} \frac{d}{dz} R_q(z)$

(iv) Average number of customers in the system, $L = L_q + \rho$

(v) Average waiting time in the system, $W = \frac{L}{\lambda_k}$

(vi) Average waiting time in the queue, $W_q = \frac{L_q}{\lambda_k}$

2.5 NUMERICAL ANALYSIS

2.5.1 Effect on Completion of Vacation $\mu_1 = 4.5,4.7,4.9,5.2,5.7$

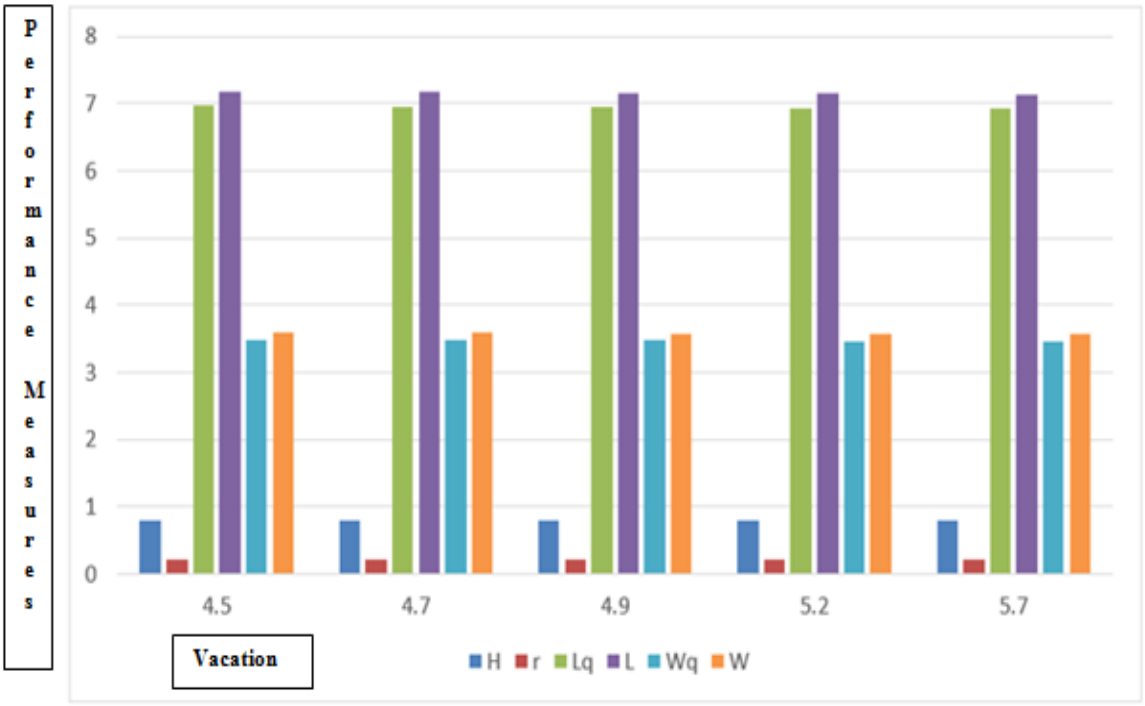

Figure 2.2 Consequence of the effect of vacation rate

Figure 2.2 shows the performance study of the vacation rate through the defined queuing system. As the maintenance work gets completed ahead of schedule during the vacation, the server quickly returns to the system and provide the service. In addition, the concept of restricted acceptance is implemented during the vacation. This also helps the system to provide services to all new customers smoothly without delay. As a result,

the system operated effectively. All the corresponding measures of the system are reduced and the customer leaves the system with fullest satisfaction after receiving the service.

CHAPTER 3

A STUDY ON A NON-MARKOVIAN BOUNDARY VALUE PROBLEM'S ORGANIZATIONAL AND QUEUE DESCRIPTION

3.1 INTRODUCTION

In a queuing system a server vacation can result from a variety of circumstances, including equipment failure, network management, and periodic workstations. In this model, the vacation is given in two stages, short vacation and long vacation. The bulk of the maintenance work will be done when the servers are on vacation. During the brief vacation time, the system's essential maintenance will be completed. The first stage of the long vacation will involve significant maintenance work. A long vacation's second phase is optional as well. Stage 2 of the long vacation will be taken into consideration if more maintenance work needs to be done on the system.

3.2 DIAGRAMMATIC REPRESENTATION OF THE MODEL

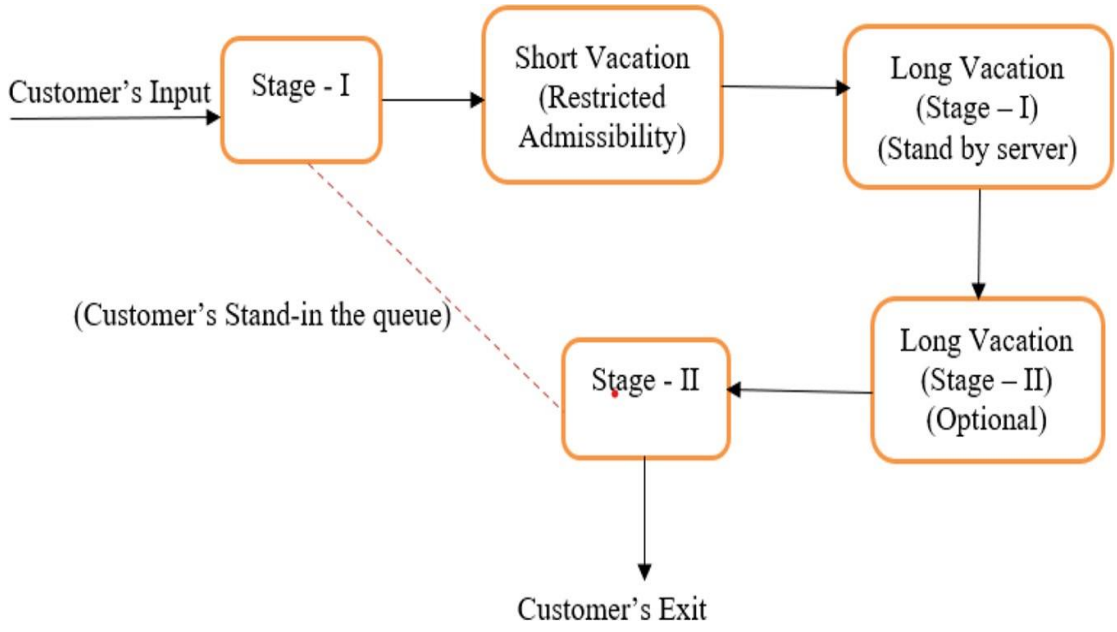
Customer's Input
Stage - I
Short Vacation (Restricted Admissibility)
Long Vacation (Stage – I) (Stand by server)
(Customer's Stand-in the queue)
Stage - II
Long Vacation (Stage – II) (Optional)
Customer's Exit

Figure 3.1 Schematic view of the queuing modeling

In this queuing problem as in Figure 3.1, Customers wait in line for the second step of service after completing the first round of administration. The server takes a brief vacation to complete the necessary server maintenance after providing the first stage of service to all arriving clients and if there are no clients waiting in line for the first level of service. The server takes a long vacation that is split into two halves after the short vacation is over. However, the first step of extended vacation is necessary, although the second stage is not. The above description of the process' maintenance requirements is in-depth. Additionally, in order to reduce lengthy waits for procedures during brief vacations, the concept of restricted admissibility is established. Additionally, a backup server is accessible in case the server takes a second stage of an extended vacation.

3.3 PERFORMANCE MEASURES OF THE QUEUING MODEL

The queuing system's supplementary variable technique is used to address the queuing problem. Using this technique the probability generating function of the queue size is derived.

$$R_q(z) = \left\{ \frac{\lambda_k H(D(z)-1)[\left[\frac{1-A_1^*(a)}{a}\right] + A_1^*(a)\left[\frac{1-Y_1^*(c)}{c}\right] + A_1^*(a)Y_1^*(c)\left[\frac{1-Y_2^*(e)}{e}\right] + A_1^*(a)Y_1^*(c)Y_2^*(e)\left[\frac{1-Y_3^*(a)}{a}\right] + A_1^*(a)Y_1^*(c)Y_2^*(e)Y_3^*(a)\left[\frac{1-A_2^*(a)}{a}\right]]}{z - A_1^*(a)Y_1^*(c)Y_2^*(e)A_2^*(a)[mY_3^*(a) + (1-m)]} \right\}$$

Using Little's formula the other performance measures are derived.

(i) Idle time of the server $H = 1 - R_q(1)$

(ii) Utilization factor $\rho = 1 - H$

(iii) Average number of customers in the queue $L_q = \lim_{z \to 1} \frac{d}{dz} R_q(z)$

(iv) Average number of customers in the system, $L = L_q + \rho$

(v) Average waiting time in the system, $W = \frac{L}{\lambda_k}$

(vi) Average waiting time in the queue, $W_q = \frac{L_q}{\lambda_k}$

3.4 NUMERICAL STUDY

3.4.1 Effect of Restricted admissibility $p = 0.8, 1.2, 1.6, 2, 2.4$

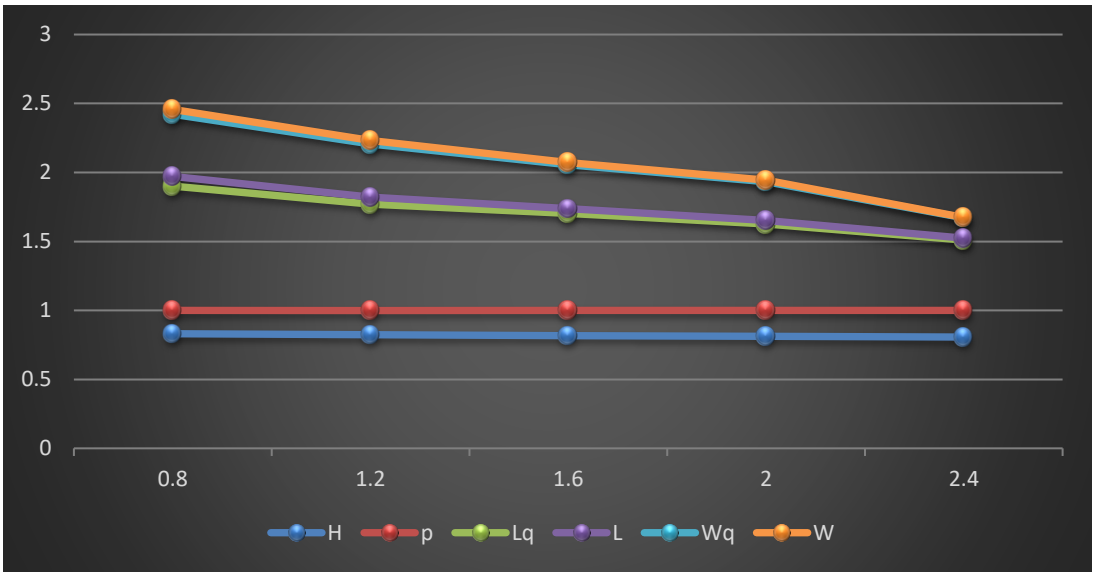

Figure 3.2 Graphical view of the effect of Restricted admissibility

Due to maintenance tasks completed over the brief vacation, there is a restricted admittance. Figure 3.2 demonstrates that line length and wait time reduce as restricted admissibility rises. The amount of idle time decreases if the server keeps running. The utilization factor rises as well. Admission is prohibited during brief breaks. Hence, the queue's length is decreased.

CHAPTER 4

ASSESSMENT OF PERFORMANCE SIMULATION IN VLSI TESTING PROCEDURES

4.1 INTRODUCTION

In order to improve the quality of devices before they are delivered to customers, VLSI testing processes have been created to detect damaged devices using

automated test equipment. To characterize a chip and identify manufacturing issues that must be resolved before full production, a variety of devices must be tested. This testing methodology for services includes a time-consuming and error-prone planning process. Through the use of a queuing strategy, the performance level of the process used in these testing approaches are investigated and examined.

4.2 QUEUING STRATEGY IN VLSI TESTING METHODOLOGY OF SERVICES

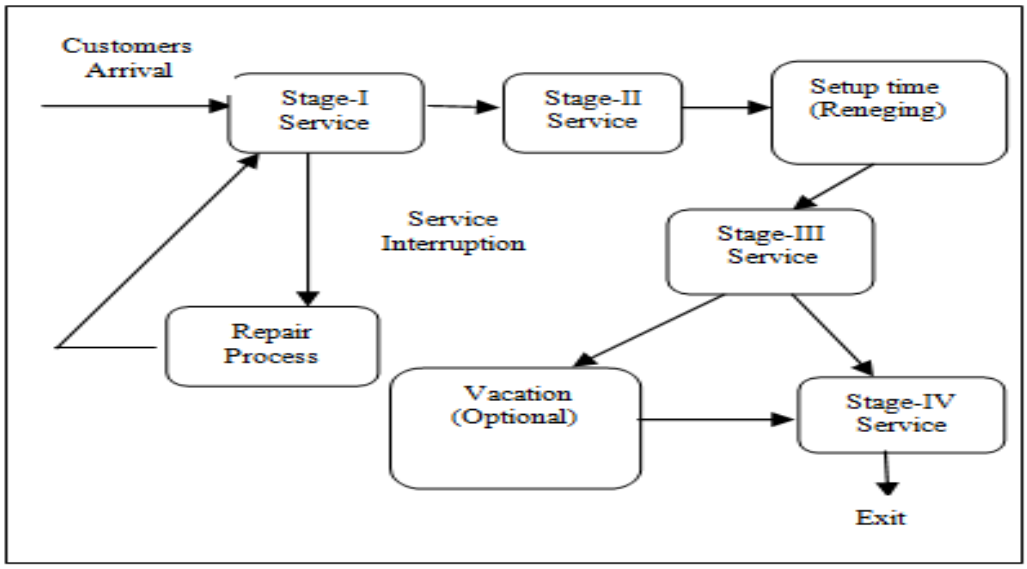

Figure. 4.1 Queuing model of the process in VLSI testing

In this testing process as in Figure 4.1, all the devices are given as input. Next in Stage 1 DC Parametric test is conducted, interruptions arises during this test and the repair process is carried out immediately. Next AC Parametric test is conducted in Stage 2. When the defective component was being tested and sometimes it takes longer time. Due to that other devices kept in the line to test the components are removed from the process. This situation is termed to Reneging. Here, a structural test approach with a stuck at fault model is used. Particularly this happens during the process of AC Parametric test as it takes longer time compared to the other tests. Following, setup time stage takes place before the actual happening of functional test, the basic characterization is to be carried out in prior. Next, In Stage 3 Functional tests are conducted. After the completion of stage 3 Optional

vacation takes place here vacation means maintenance work which helps the system is detect the flaws or to minimize the interruption which arises during the time of service it is called Burn in test. Finally Productivity test in Stage 4 takes place and the process gets completed.

4.3 PERFORMANCE MEASURES OF THE QUEUING MODEL

The above queuing problem is approached through supplementary variable technique and all the corresponding performance measures are derived.

The probability generating function of the queue size is

$$R_q(z) = \frac{\lambda_k H[D(z)-1]\left[\frac{1-A_1^*(b)}{b}\right]\left\{1+\sigma z A_2^*(a)\left[\begin{matrix}1+B^*(a)+B^*(a)S_1^*(c)+B^*(a)S_1^*(c)A_3^*(a)+\\ mB^*(a)S_1^*(c)A_3^*(a)Y_1^*(a)\left(1+A_4^*(a)\right)\end{matrix}\right]\right\}}{z-\sigma z A_2^*(a)\left[\frac{1-A_1^*(b)}{b}\right]\left(1+mB^*(a)S_1^*(c)A_3^*(a)Y_1^*(a)A_4^*(a)\right)}$$

Using Little's formula the other performance measures are derived.

(i) Idle time of the server $H = 1 - R_q(1)$

(ii) Utilization factor $\rho = 1 - H$

(iii) Average number of customers in the queue $L_q = \lim_{z\to 1}\frac{d}{dz}R_q(z)$

(iv) Average number of customers in the system, $L = L_q + \rho$

(v) Average waiting time in the system, $W = \frac{L}{\lambda_k}$

(vi) Average waiting time in the queue, $W_q = \frac{L_q}{\lambda_k}$

4.4 SIMULATION ANALYSIS

4.4.1 The Impact of Completion of Setup Time Stage $\beta_1 = 5,6,7,8,9$

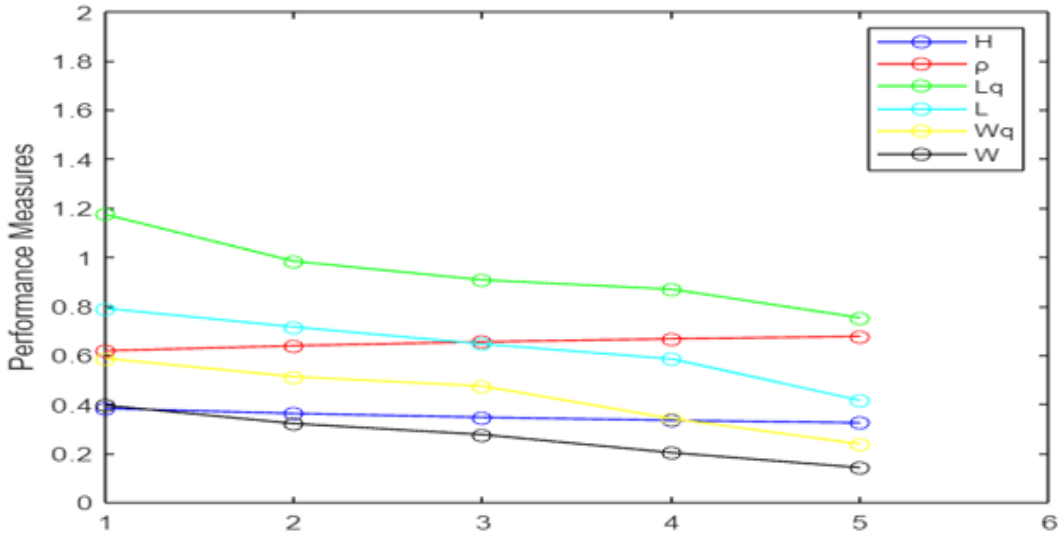

Figure 4.2 The Impact of Completion of Setup Time

Figure 4.2 demonstrates how the server has been operating faster than the utilization factor while simultaneously prolonging the waiting time following setup. The size of the queue and the waiting durations will both go down once the server has concluded the setup process.

CHAPTER 5

CONCLUSION

The probability generating function of the queue size is derived for each of the models stated above. Mean queue size, mean waiting time in the line, waiting time of customers in the system, and number of customers in the system have all been calculated as performance measurements. All of the models presented above have real-world applications. A numerical demonstration has been provided to justify the model. These findings provide extensive information that engineers, system designers, system analysts, communication networks, and other sectors may utilize to make judgments on queued systems. It leads to the required level and quality of service.

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